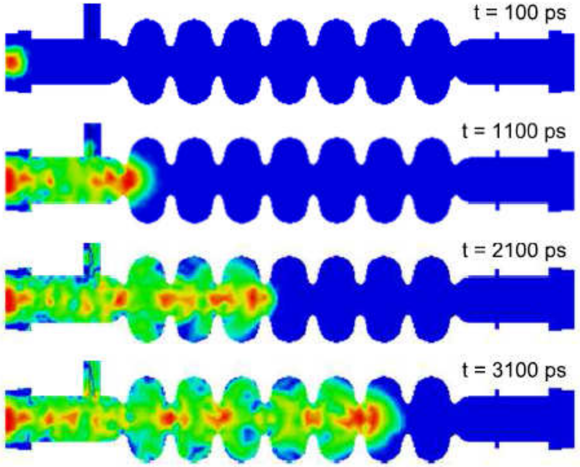
Cornell Laboratory for Accelerator-based Sciences and Education (CLASSE)

BROOKHAVEN NATIONAL LABORATORY

Beam BreakUp (BBU) Physics and Simulation

-- Interactions between higher-order modes (HOMs) and beam bunches leading to instabilities --



Jim Crittenden
William Lou

Outline

1) BBU overview

- What is BBU?
- BBU theory
- BBU simulation on Bmad

2) CBETA BBU simulation results

- What is CBETA
- Randomized HOM assignment
- I_{th} statistics (1-pass and 4-pass)

Optional) Aim for higher I_{th}

- Results with additional phase-advances
- Results with x-y coupling

Beam breakup Instability (BBU)

• Higher Order Modes (HOM) in the cavities give undesired kick.
• Off-orbit bunch returns to the cavities and excite more HOMs…(positive feedback)

• BBU limits the maximum achievable current in an ERL $\Longrightarrow$ threshold current I_{th}

• The goal is to find the I_{th} for a given lattice.

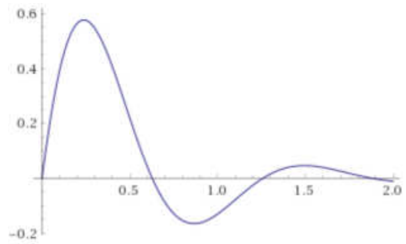
BBU theory: the mathematics

Elementary case: 1-dipole-HOM + 1 recirculation pass

$$\underbrace{\Delta p_x(t)}_{\text{Transverse momentum gained from HOM}} = \frac{e}{c} \underbrace{W(t - t')}_{\text{Wake function [N/C^2]}} \underbrace{x(t')}_{\text{Transverse offset}} \underbrace{I(t')}_{\text{Current}} dt'$$

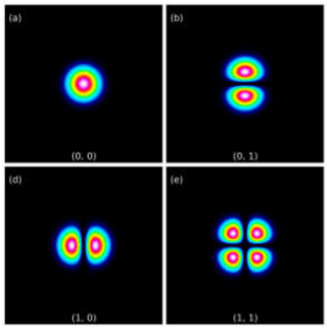
Wake function (Cavity HOM property)

$$W(\tau) = \left(\frac{R}{Q}\right)_\lambda \frac{\omega_\lambda^2}{2c} e^{-(\omega_\lambda/2Q_\lambda)\tau} \sin\omega_\lambda \tau$$



An HOM is characterized by

$$\left(\left(\frac{R}{Q}\right)_\lambda, Q_\lambda, \omega_\lambda \right)$$



HOM voltage $V(t) = \frac{c}{e} \Delta p_x(t)$

Transverse offset after recirculation $x(t + t_r) = T_{12} p_x(t)$ **T12 of the transfer matrix is a lattice property**

$$V(t) = \int_{-\infty}^{t} W(t - t')I(t')T_{12}\frac{e}{c}V(t' - t_r)dt'$$

Measured current

Bunch time spacing

Current as a train of (dirac) beam bunches $I(t) = I_0 t_b \sum_{m=-\infty}^{\infty} \delta_D(t - t_r - m t_b)$

$$V(nt_b + t_r) = I_0 t_b T_{12} \frac{e}{c} \sum_{m=0}^{\infty} W(mt_b) V([n-m]t_b)$$

To solve this difference equation, we write HOM voltage (retaining all possible frequencies ω) as:

$$V(t) = \frac{1}{2\pi} \int_{-\infty - ic_0}^{\infty - ic_0} \tilde{V}(\omega') e^{-i\omega' t} d\omega'$$

After some math, we obtain the dispersion relation between I_0 and ω

$$\boxed{\frac{1}{I_0} = t_b T_{12} \frac{e}{c} e^{i\omega n_r t_b} w(\delta)}$$

$$w(\delta) = \sum_{n=0}^{\infty} W([n+\delta]t_b) e^{i\omega n t_b}$$

At a given I_0 , multiple ω (complex) can satisfy the relation.
At a stable I_0 , all ω must have negative imaginary part.
At the threshold current, one ω crosses the real axis!!

1-pass 1-HOM thin-lens cavity

(1) General analytic formula

$$w(\delta) = \sum_{n=0}^{\infty} W([n+\delta]t_b)e^{i\omega n t_b}$$

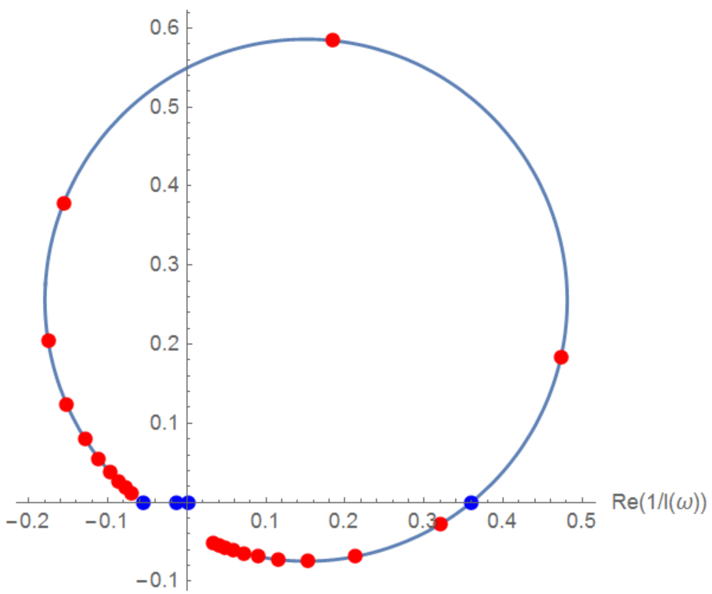
$$W(\tau) = \left(\frac{R}{Q}\right)_\lambda \frac{\omega_\lambda^2}{2c} e^{-(\omega_\lambda/2Q_\lambda)\tau} \sin\omega_\lambda \tau$$

$$\frac{1}{I_0} = t_b T_{12} \frac{e}{c} e^{i\omega n_r t_b} w(\delta)$$

Find the critical $\omega \in [0, \pi/t_b]$
which gives the maximum real I_0^{-1}

The corresponding I_0 is I_{th}

Parametric plot of 1/I(ω) around the ω with the largest Re(1/I(ω))



Analytic formulas for I_{th}

(Under different physical assumptions)

1. General $$\frac{1}{I_0} = t_b T_{12} \frac{e}{c} e^{i\omega n_r t_b} w(\delta)$$

2. Linearized $$\frac{1}{I_0} = -\frac{\mathcal{K}}{2} \frac{T_{12}\, e^{i\omega t_r}}{\Delta\omega t_b + i\epsilon}$$

3. Approximate $$I_{\text{th}} = -\frac{\epsilon}{\mathcal{K}} \frac{2}{T_{12} \sin\omega_\lambda t_r} \qquad T_{12} \sin\omega_\lambda t_r < 0$$

$$I_{\text{th}} = \frac{2}{\mathcal{K}|T_{12}|} \sqrt{\epsilon^2 + \frac{1}{n_r^2} \text{mod}(\omega_\lambda t_r, \pi)^2} \qquad T_{12} \sin\omega_\lambda t_r > 0$$

1-pass 1-HOM thin-lens cavity

(2) Linearized analytic formula

Valid only if $\epsilon \ll 1$

HOM decay is negligible on the time scale of bunch spacing

$$\frac{1}{I_0} = -\frac{\mathcal{K}}{2}\frac{T_{12}\,e^{i\omega t_r}}{\Delta\omega t_b + i\epsilon}$$

$$\epsilon = (\omega_\lambda/2Q_\lambda)t_b$$

$$\mathcal{K} = t_b(e/c^2)(R/Q)_\lambda(\omega_\lambda^2/2)$$

$$\Delta\omega = \omega - \omega_\lambda$$

Same method to find I_{th} as case 1

1-pass 1-HOM thin-lens cavity

Approximate analytic formula

Valid only if $n_r \epsilon \ll 1$

$$n_r = Top(t_r/t_b)$$

$$\epsilon = (\omega_\lambda/2Q_\lambda)t_b$$

HOM decay is negligible on the time scale of recirculation

For $T_{12}\sin\omega_\lambda t_r < 0$

$$I_{\text{th}} = -\frac{\epsilon}{\mathcal{K}}\frac{2}{T_{12}\sin\omega_\lambda t_r}$$ (the trough)

$$\boxed{= -\frac{2c^2}{e(\frac{R}{Q})_\lambda Q_\lambda \omega_\lambda}\frac{1}{T_{12}\sin\omega_\lambda t_r}}$$

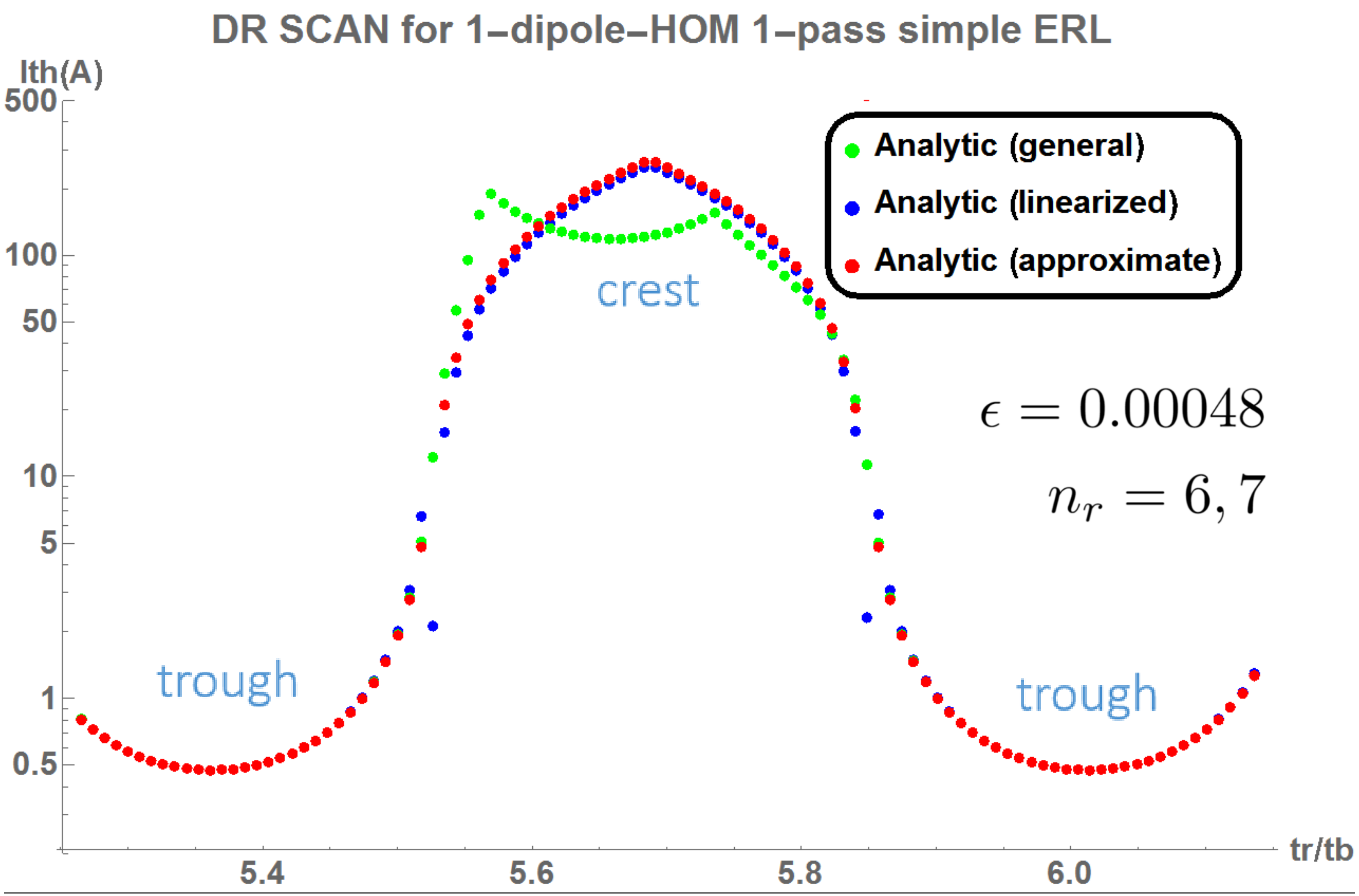
$T_{12}\sin\omega_\lambda t_r > 0$

$$I_{\text{th}} = \frac{2}{\mathcal{K}|T_{12}|}\sqrt{\epsilon^2 + \frac{1}{n_r^2}\text{mod}(\omega_\lambda t_r, \pi)^2}$$ (the crest)

Use the famous formula with caution

Make sure the physical condition is met

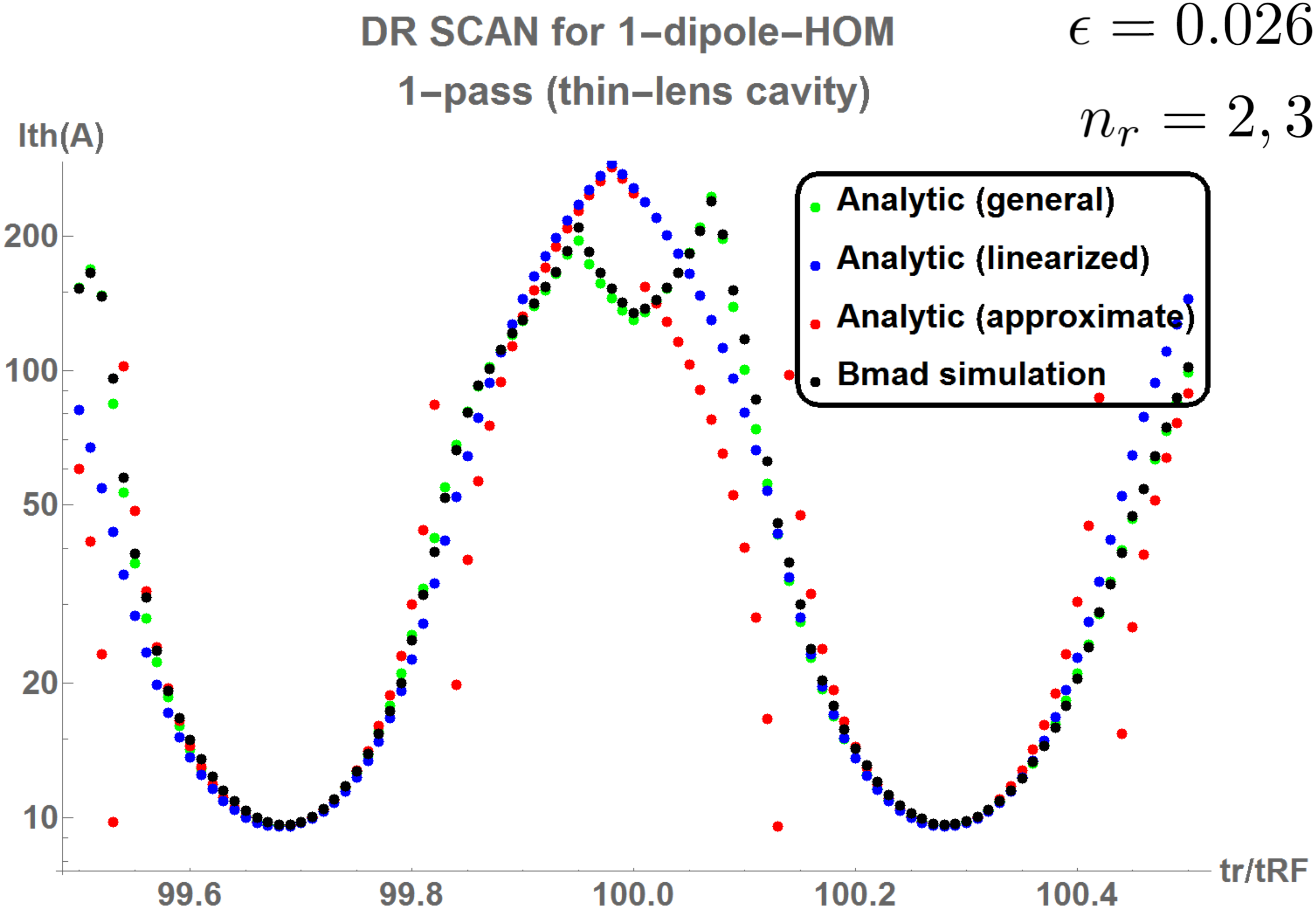
1-pass 1-HOM thin-lens cavity



Theory v.s simulation

$$\epsilon = 0.026$$

$$n_r = 2, 3$$



BBU theory

	Number of recirculation pass	Number of HOMs	Comments
Case 1	1	1	- Most elementary BBU model - A few analytic formulas available for I_{th}
Case 2	Np > 1	1	- An intermediate case - A linearized analytic formula available
Case 3	Np > 1	N > 1	- A general case - Difficult to apply analytic formula - Simulation required to find I_{th}

Current BBU theory (all 3 cases) assumes
all HOM(s) are dipoles, and the cavity(s) are thin-lens

Case 2

Formulas for “Np-pass 1-HOM” (Np>1) ?

(1) General: (Difficult) Find the maximum real eigenvalue of

$$\frac{e}{c} t_b \sum_{I=J+1}^{N_p} w(\delta(I,L)) e^{i\omega \mathrm{Top}[(t^I - t^L)/t_b] t_b} T^{IJ} \qquad \omega \in [0, \pi/t_b]$$

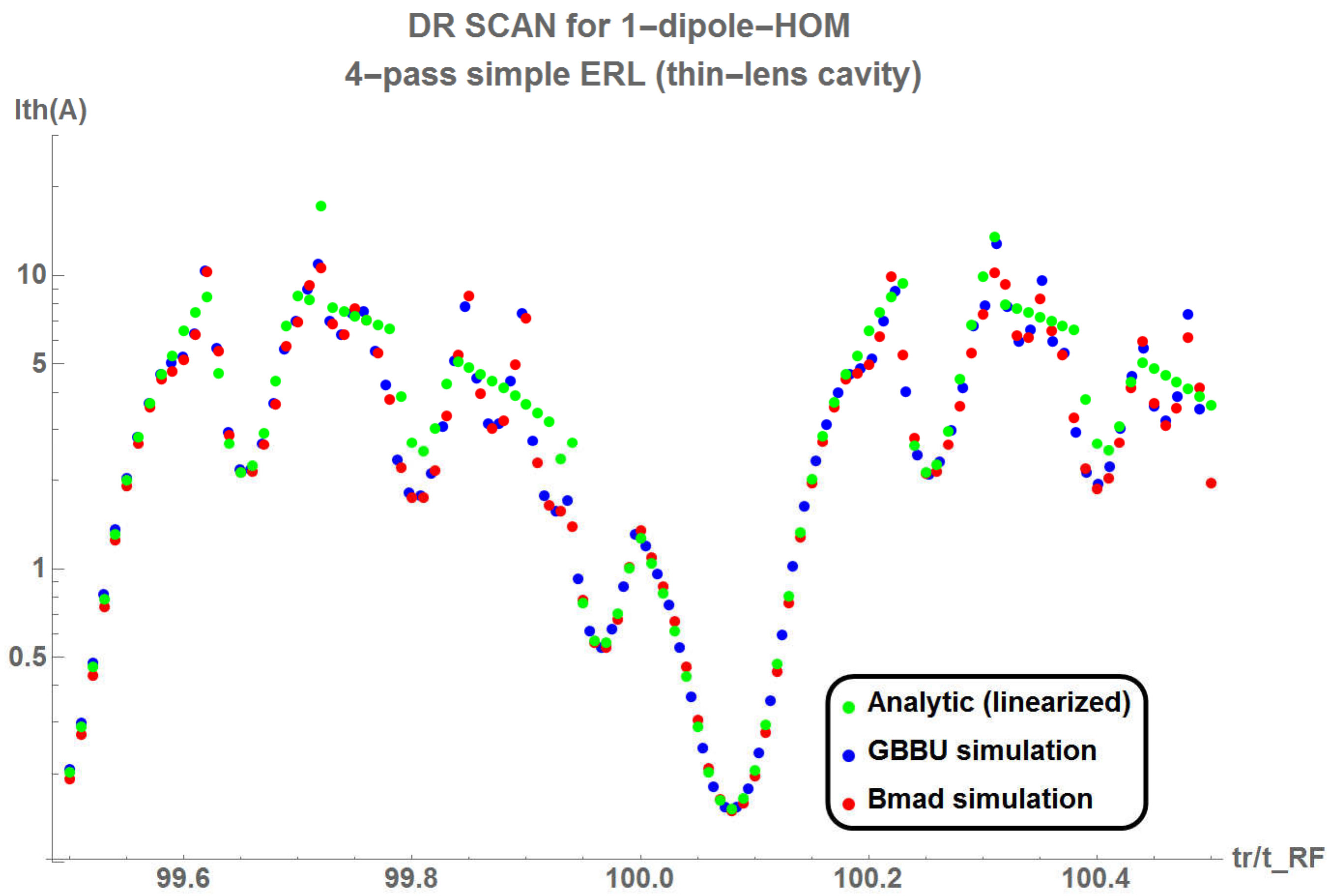
(2) Linearized: (straightforward) Find the maximum real value of

$$\frac{1}{I_0} = -\frac{\mathcal{K}}{2} \frac{1}{\Delta\omega t_b + i\epsilon} \sum_{J=1}^{N_p} \sum_{I=J+1}^{N_p} e^{i\omega(t^I - t^J)} T^{IJ}$$

(3) Approximate: N/A

Case 2: Np-pass 1-HOM

Theory v.s simulation



For a 4-pass ERL with 1 HOM, simulation well agrees with the theory (linearized analytic formula), except on the "crests"

Case 3

General formula (Np-pass, N-cav) for I_{th}

$$\frac{1}{I_0}\vec{V} = \mathbf{M}(\omega)\vec{V}$$

$$w(\delta) = \sum_{n=0}^{\infty} W([n+\delta]t_b)e^{i\omega n t_b}$$

$$W(\tau) = \left(\frac{R}{Q}\right)_\lambda \frac{\omega_\lambda^2}{2c} e^{-(\omega_\lambda/2Q_\lambda)\tau} \sin\omega_\lambda \tau$$

$$M_{ij}^{LJ} = \frac{e}{c} t_b \sum_{I=J+\Theta_{j,i}}^{N_p} w_i(\delta(I,L)) e^{i\omega \mathrm{Top}[(t^I - t^L)/t_b] t_b} T_{ij}^{IJ}$$

Find the critical $\omega \in [0, \pi/t_b]$
which gives the maximum real eigenvalue of M

The corresponding I_0 is I_{th}

This is numerically difficult

Summary on BBU theory

- For case 1 (1-pass, 1-HOM), the famous formula

$$I_{\text{th}} = -\frac{2c^2}{e(\frac{R}{Q})_\lambda Q_\lambda \omega_\lambda} \frac{1}{T_{12}\sin\omega_\lambda t_r}$$ is only an approximation.

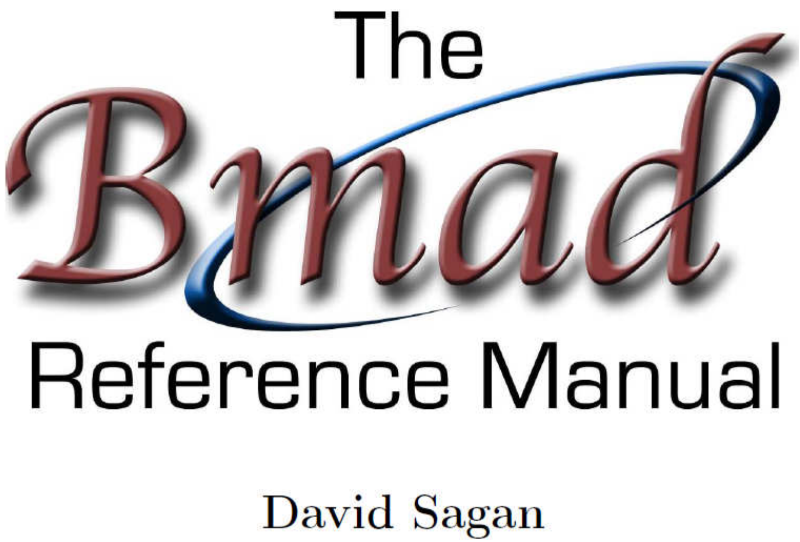
Use the general formula whenever possible.

- For case 2 (Np-pass, 1-HOM), a linearlized analytic formula has been checked with simulation.

- For case 3 or even more general cases (with coupling or HOMs of higher order), a stronger numerical method is required to apply the general formula.

Questions?

Bmad software



- Developed by Cornell LEPP
- Open source, free, compiled in C and Fortran

- Multi-pass lattice design, lattice optimization, multi-particle tracking algorithms, wakefields, Taylor maps, real-time control “knobs”…
- Constantly (daily to weekly) updated
- https://www.classe.cornell.edu/~dcs/bmad/overview.html

BBU simulation on Bmad

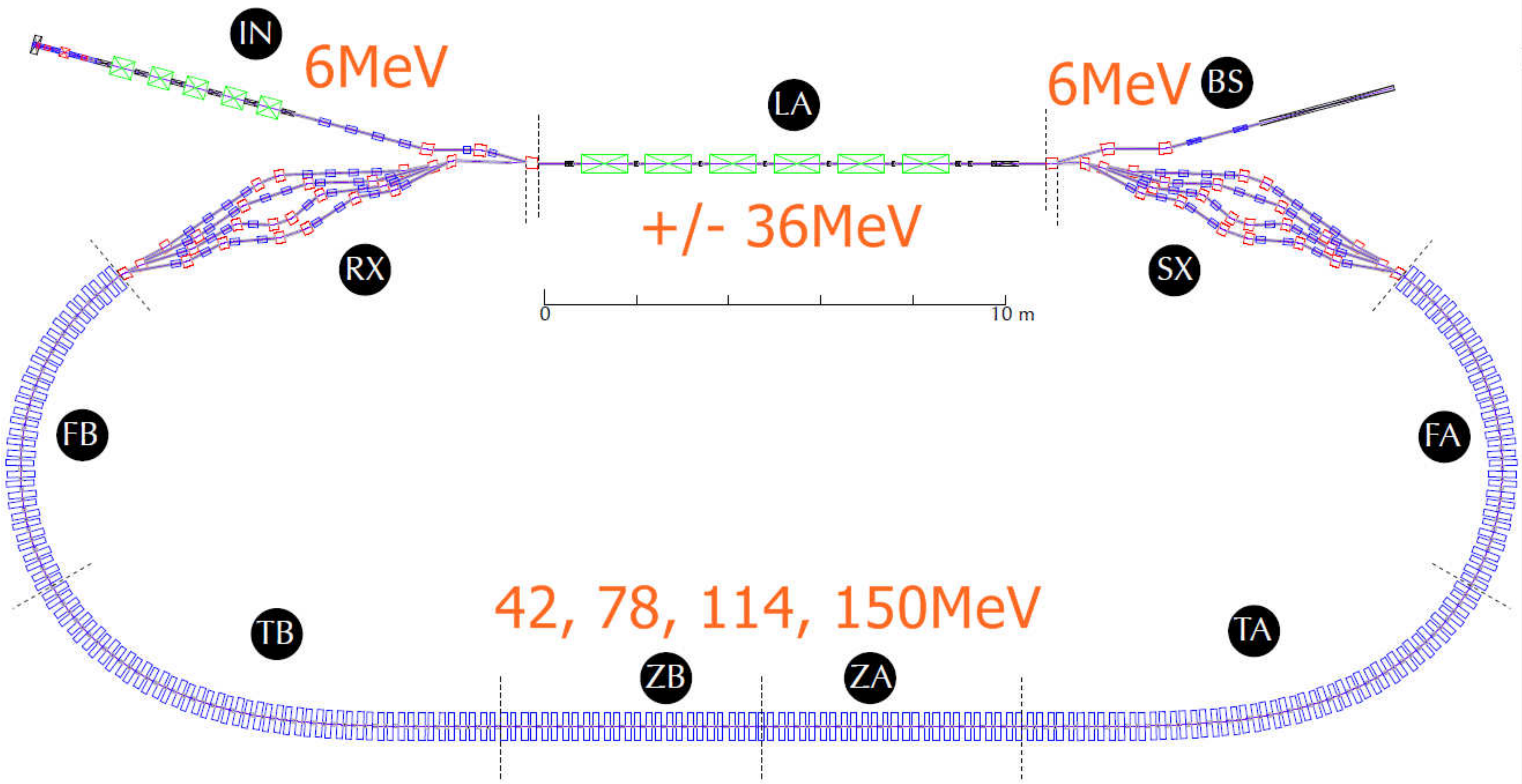
- Given a complete lattice with multi-pass cavities and HOMs assigned…

- Starts with a test current…

1. tracks off-orbit bunches through lattice
2. computes bunch-HOM momentum exchanges
3. determines stability of all HOM voltages

- Attempts different test currents to pin down I_{th}

CBETA


IN
6MeV
LA
6MeV
BS
+/- 36MeV
RX
SX
0
10 m
FB
FA
42, 78, 114, 150MeV
TB
TA
ZB
ZA

Design current of CBETA

Design current (mA)	KPP	UPP
1-pass	1	40
4-pass	-	40

Simulated HOM data for one CBETA cavity


```
                 Frequency R/Q          Q        mode  Polarization_Angle
                 (Hz)      Ohm/m^(2n)                  (Radians/2pi)
&long_range_modes
   lr(1) =       8.8302e9  7765.5       606830. 1      0.
   lr(2) =       3.0751e9  3901.5       310240. 1      0.
   lr(3) =       2.549e9   81610.       6229.9  1      0.
   lr(4) =       1.7041e9  51754.       1654.5  1      0.
   lr(5) =       1.7381e9  42511.       1755.8  1      0.
   lr(6) =       1.8702e9  39137.       1610.   1      0.
   lr(7) =       1.8558e9  25852.       1598.9  1      0.
   lr(8) =       1.8711e9  42890.       789.99  1      0.
   lr(9) =       1.872e9   40762.       653.48  1      0.
   lr(10) =      1.6766e9  11687.       707.34  1      0.
/
```

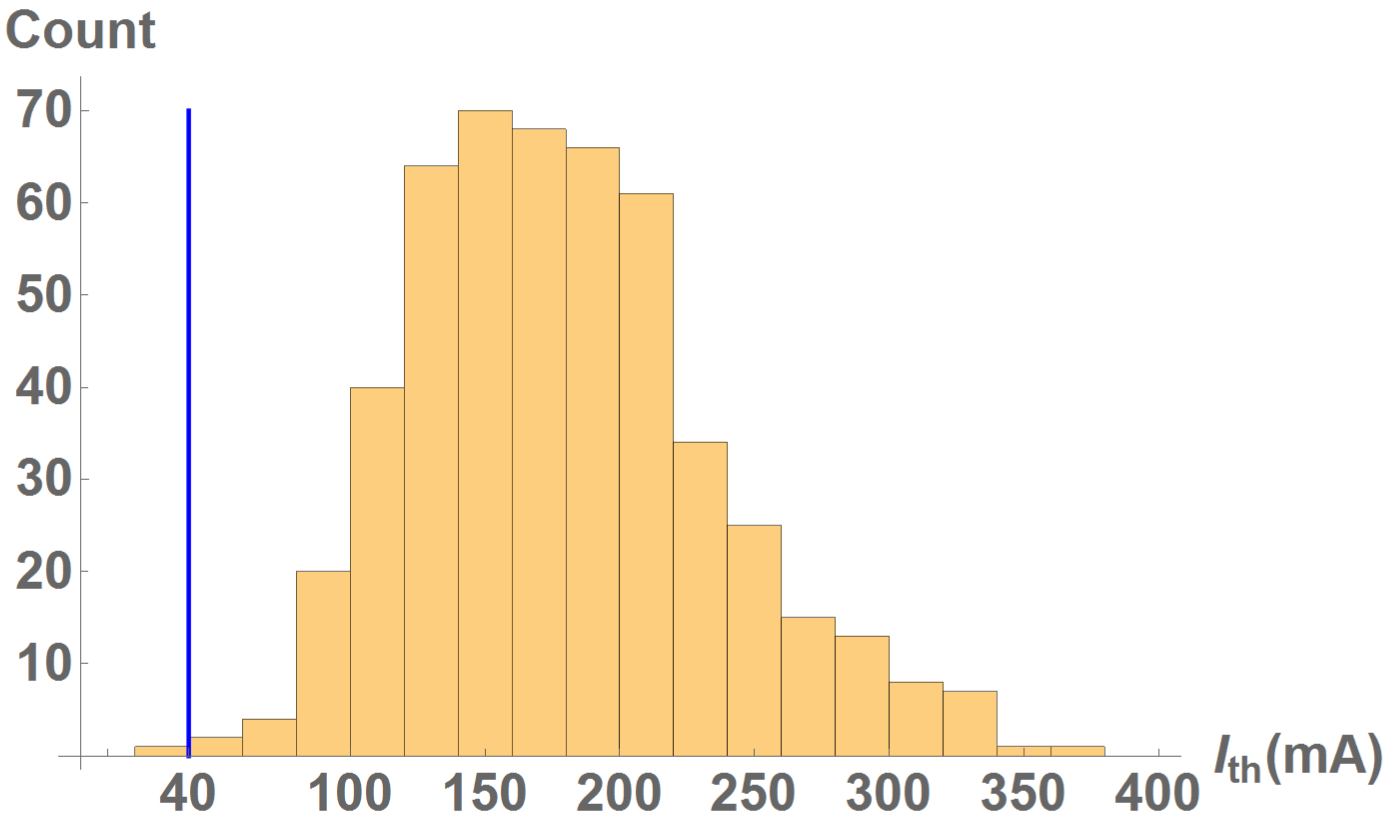
- Cavity construction error: ± 125 μm, 250 μm…
- 400 unique cavities provided per error case.

The “10 worst dipole HOMs” (large figure of merit) provided per cavity.

$$\xi_\lambda = (R/Q)_\lambda \frac{\sqrt{(Q_L)_\lambda}}{f_\lambda}$$

CBETA 1-pass

Cavity shape error: 125 μm
HOM assignment: random (10 dipole/cavity)

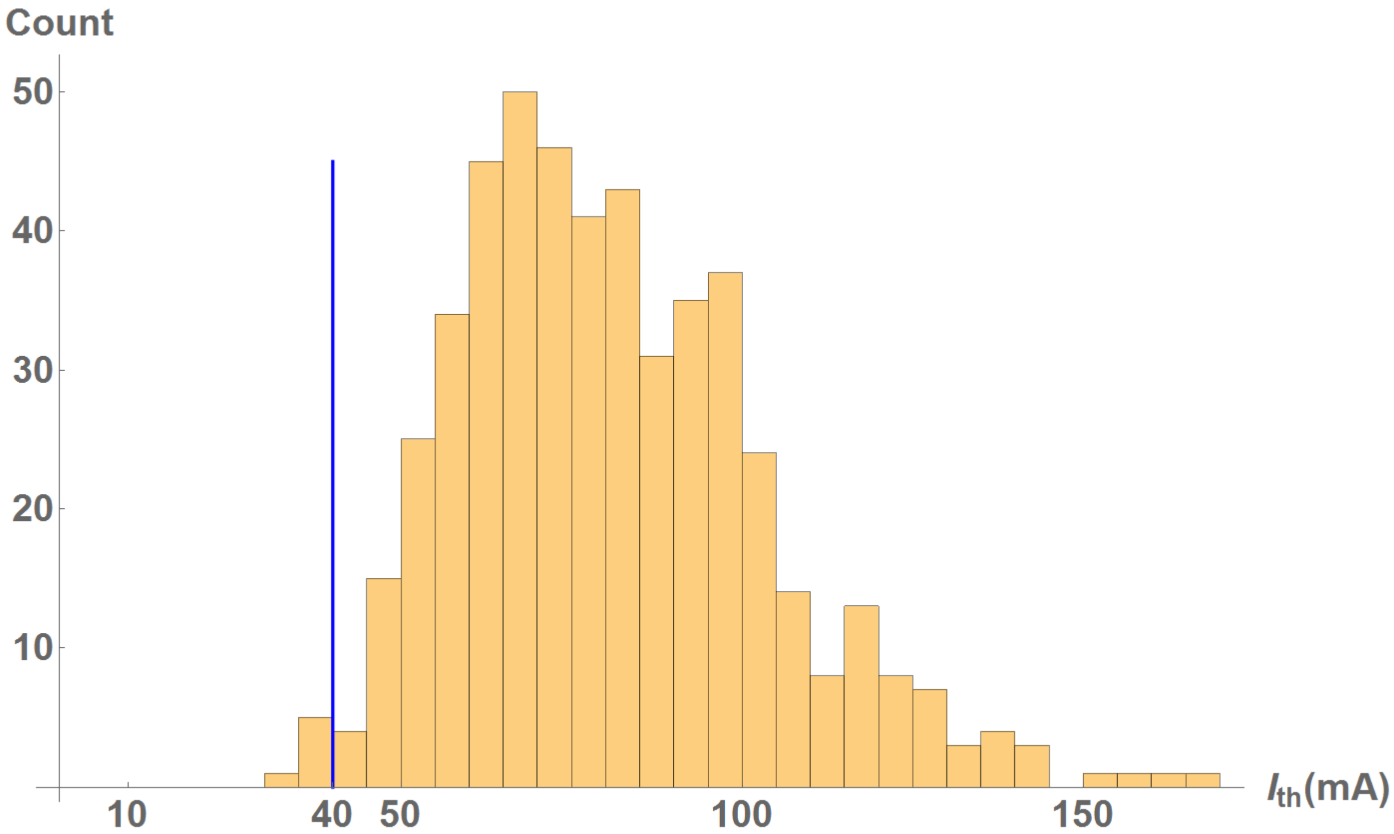


All I_{th} results above 30 mA

1 out of 500 below 40mA

CBETA 4-pass

Cavity shape error: 125 μm
HOM assignment: random (10 dipole/cavity)

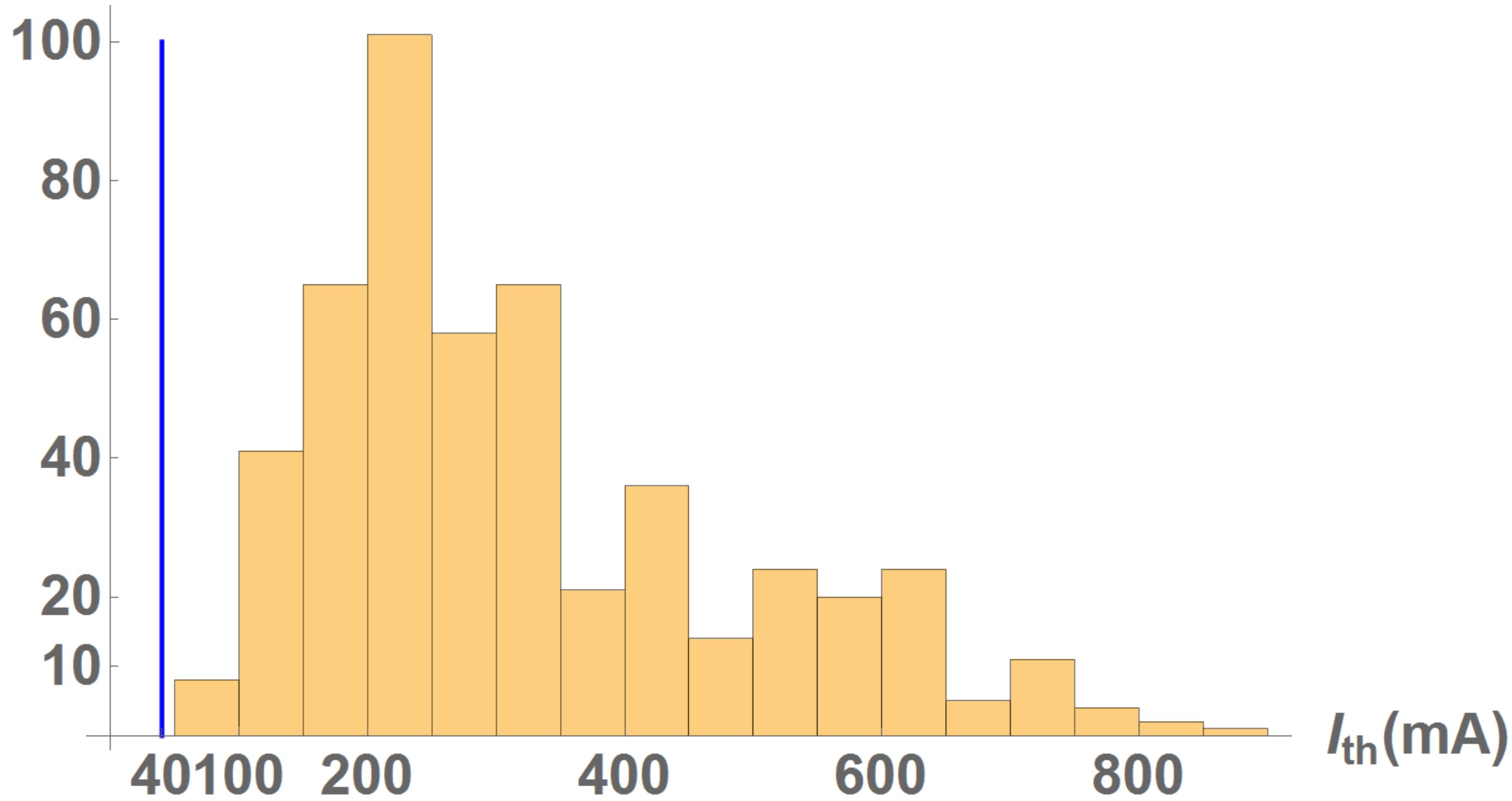


All I_{th} results above 10 mA

<u>6 out of 500</u> below 40mA

CBETA 4-pass

Cavity shape error: 250 μm
HOM assignment: random (10 dipole/cavity)



All $I_{\rm th}$ results above 40mA

Questions?

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Aim for better I_{th}

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$$M_{ij}^{LJ} = \frac{e}{c} t_b \sum_{I=J+\Theta_{j,i}}^{N_p} w_i(\delta(I,L)) e^{i\omega \mathrm{Top}[(t^I - t^L)/t_b] t_b} T_{ij}^{IJ}$$

Potential ways to improve I_{th} :

1) Change bunch injection time t_b
2) Introduce additional phase advance
3) Introduce x-y coupling

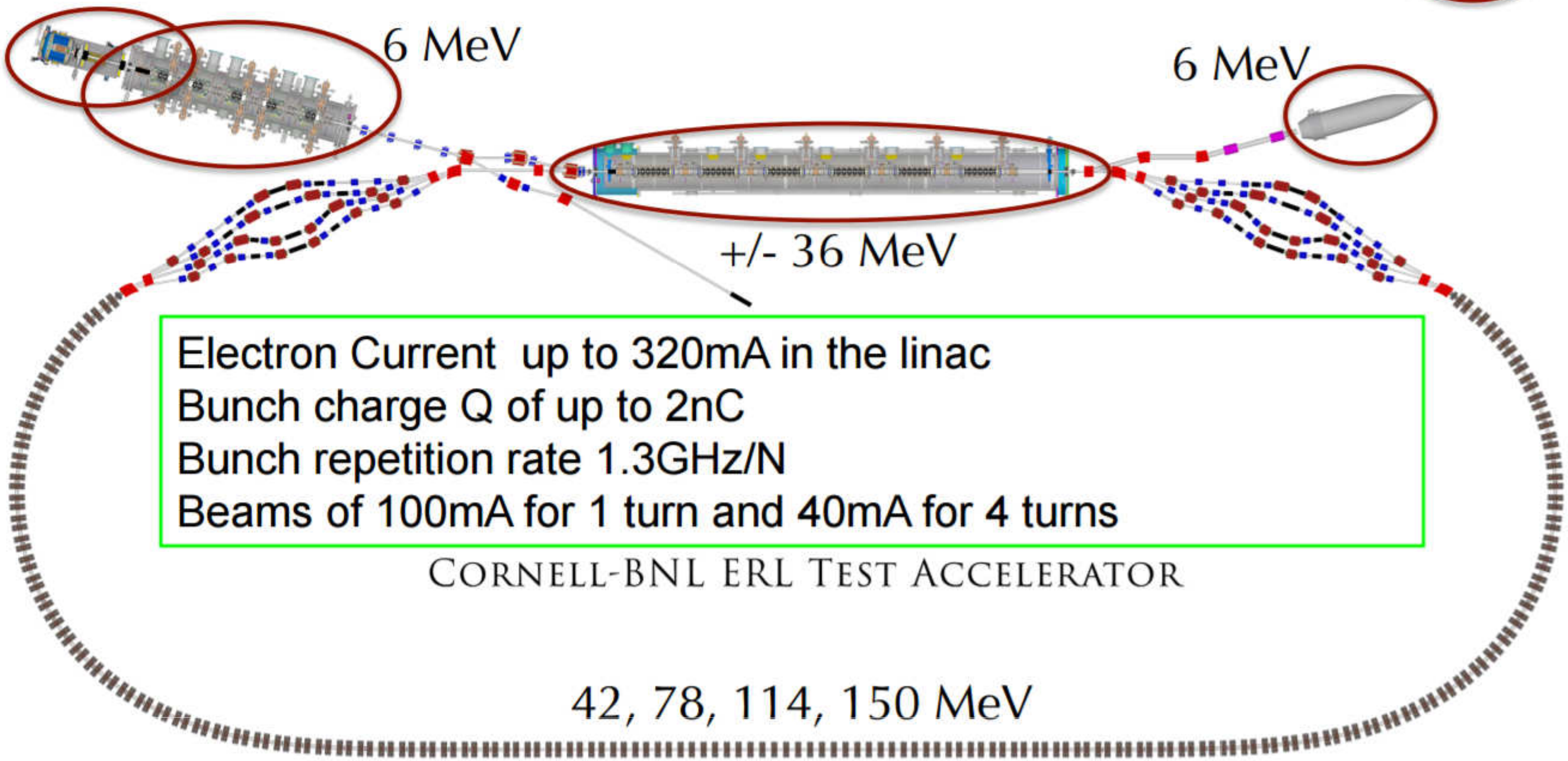
Does $I_{\rm th}$ vary with bunch frequency?

f_{RF}/f_b	4-pass $I_{\rm th}$ (mA) Averaged over 500 simulations
4	80.8
5	79.8
8	74.9
13	81.4
20	84.8
31	83.8

CBETA

- Cornell DC gun
- 100mA, 6MeV SRF injector (ICM)
- 600kW beam dump
- 100mA, 6-cavity SRF CW Linac (MLC)

Existing components at Cornell



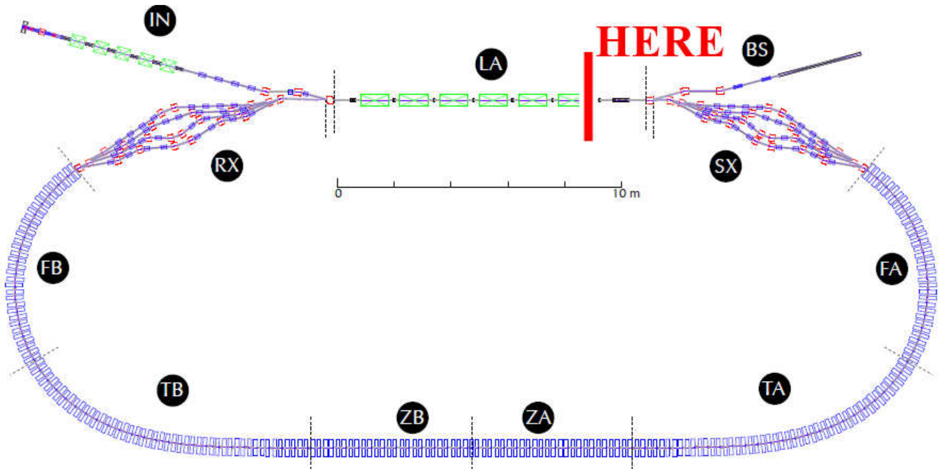
Vary the optics in Bmad

$$M_{1\leftarrow 0}(\phi) = \begin{pmatrix} \sqrt{\frac{\beta_1}{\beta_0}}(\cos\phi + \alpha_0 \sin\phi) & \sqrt{\beta_1\beta_0}\sin\phi \\ \frac{1}{\sqrt{\beta_1\beta_0}}[(\alpha_0 - \alpha_1)\cos\phi - (1+\alpha_0\alpha_1)\sin\phi] & \sqrt{\frac{\beta_1}{\beta_0}}(\cos\phi - \alpha_1\sin\phi) \end{pmatrix}$$

Two cases:

$$T_{decoupled}(\phi_x, \phi_y) = \begin{pmatrix} M_{x\leftarrow x}(\phi_x) & \mathbf{0} \\ \mathbf{0} & M_{y\leftarrow y}(\phi_y) \end{pmatrix}$$

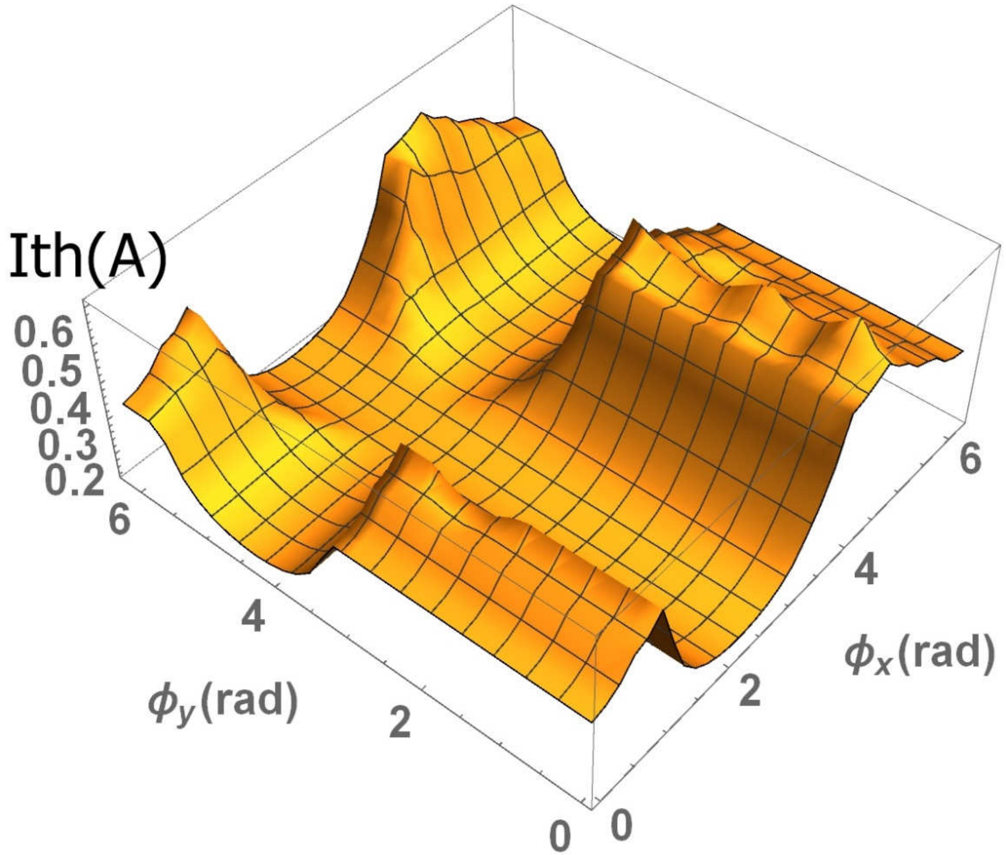
$$T_{coupled}(\phi_1, \phi_2) = \begin{pmatrix} \mathbf{0} & M_{x\leftarrow y}(\phi_1) \\ M_{y\leftarrow x}(\phi_2) & \mathbf{0} \end{pmatrix}$$



Introduce either T matrix at the end of LINAC 1st pass

CBETA 1-pass

I_{th} v.s additional phase advances (decoupled optics)



Min = 140 mA

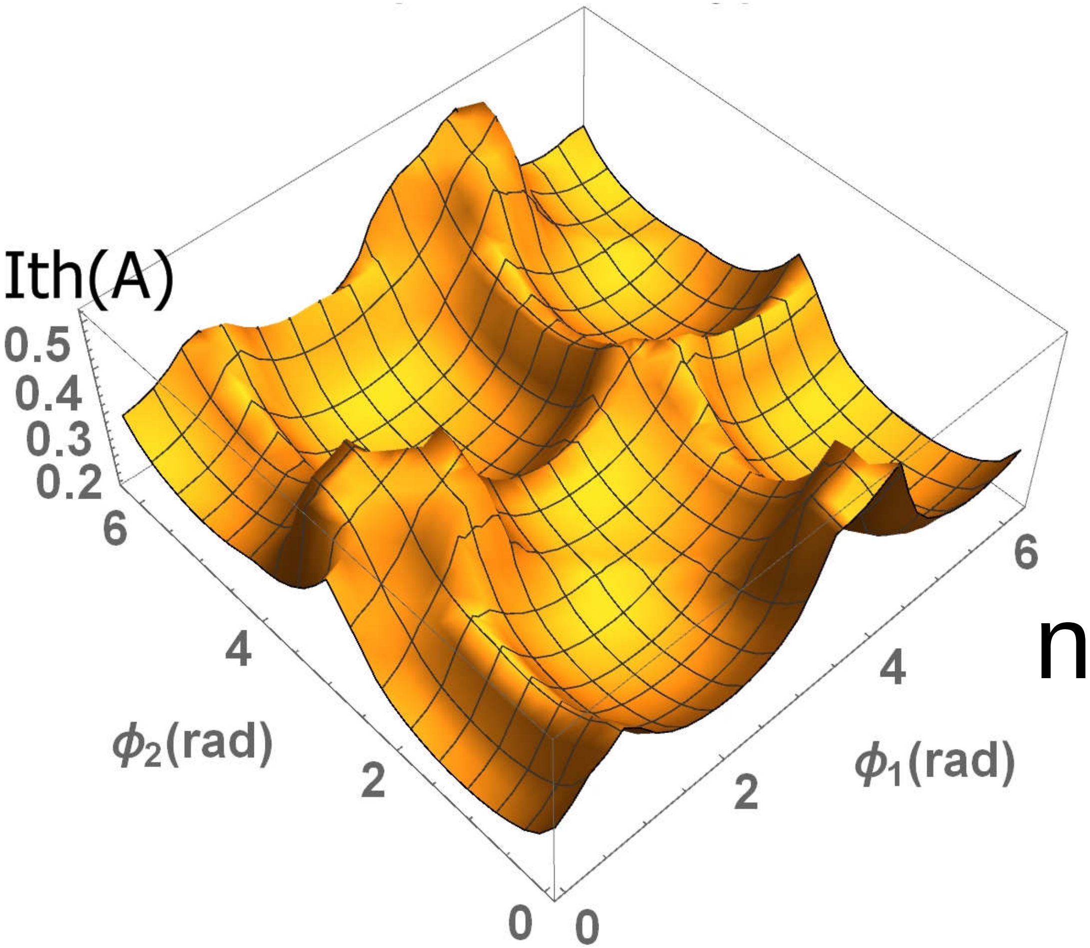
Max = 611 mA

nominal = 342 mA

I_{th} results can improve significantly

CBETA 1-pass

I_{th} with x-y coupling



Min = 140 mA

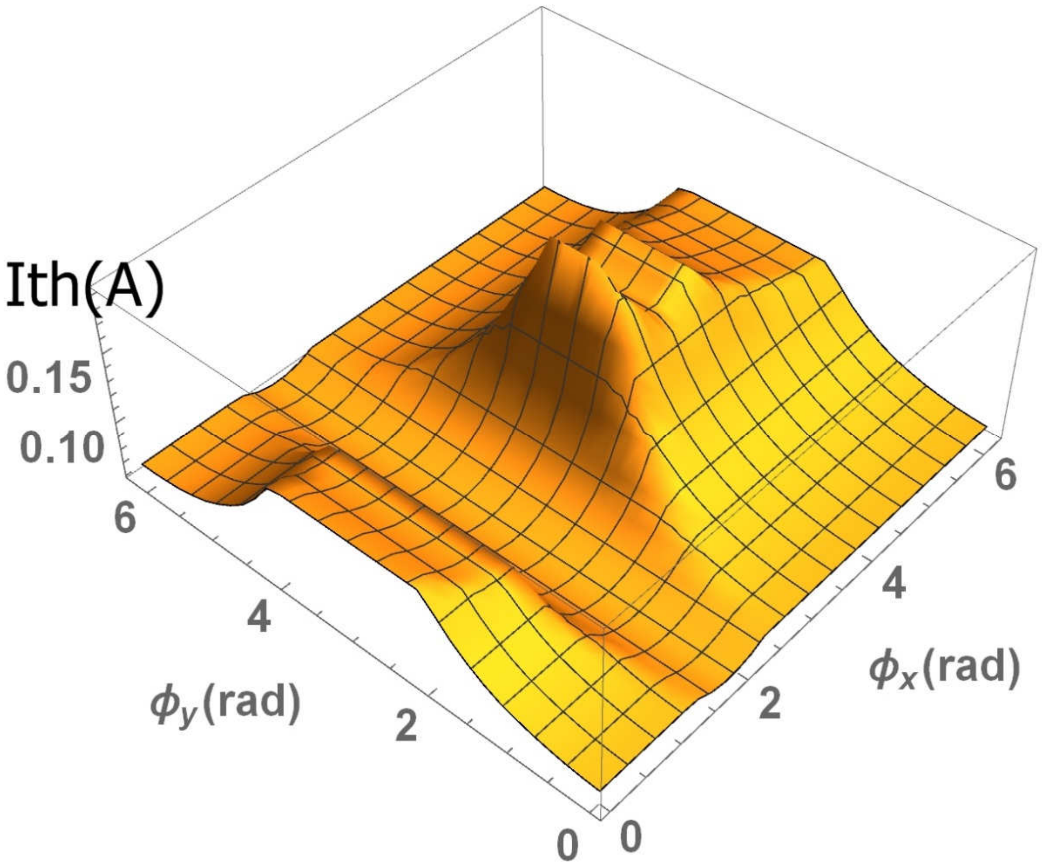
Max = 520 mA

nominal = 342 mA

I_{th} results can improve significantly

CBETA 4-pass

$I_{\rm th}$ v.s additional phase advances (decoupled optics)

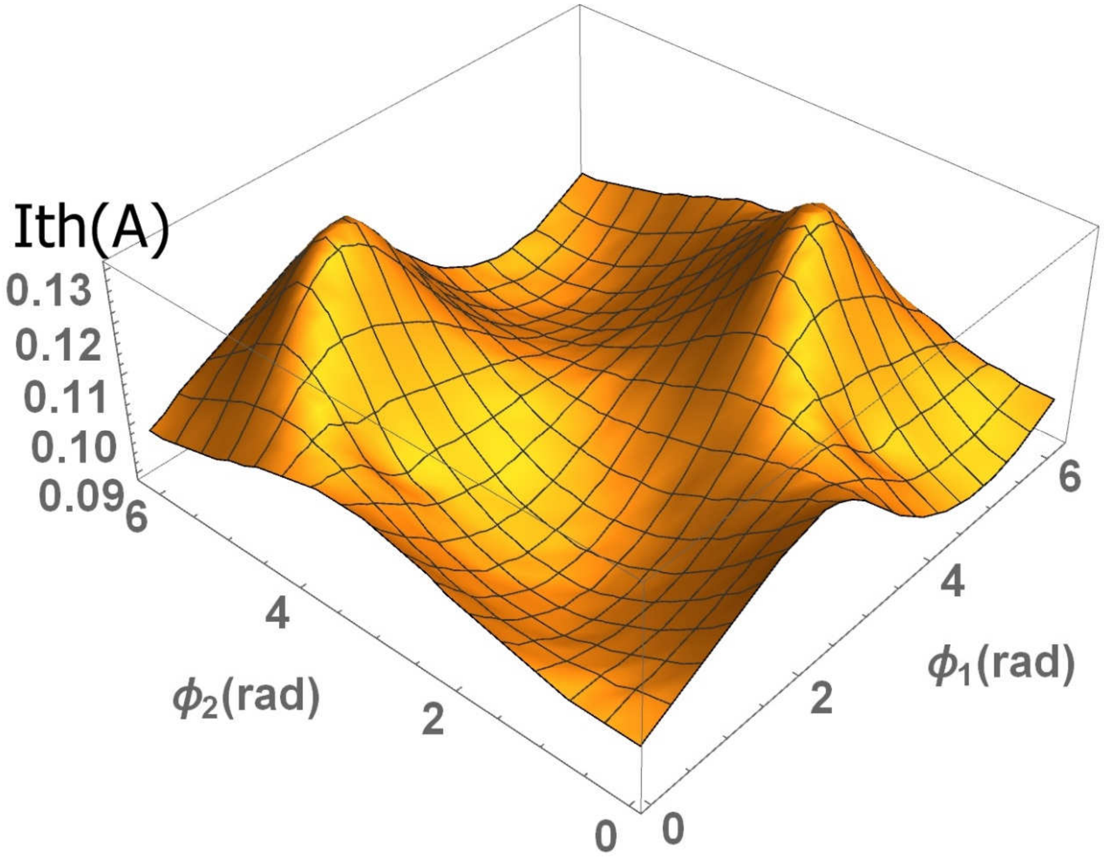


Min = 61 mA
Max = 193 mA
Nominal = 69 mA

$I_{\rm th}$ results can improve

CBETA 4-pass

$I_{\rm th}$ with x-y coupling



Min = 89 mA
Max = 131 mA
Nominal = 69 mA

$I_{\rm th}$ results can improve

Potential improvement on Ith from the current design (mA)	Additional phase advances (decoupled optics)	x-y coupling
1-pass	~ 200 mA to 400 mA	~ 200 mA to 400 mA
4-pass	~150 mA	~ 60 mA

CBETA BBU results

- For 1-pass, 99% simulated I_{th} are above the UPP (40mA)

- For 4-pass, 98% simulated I_{th} are above the UPP (40mA)

- For 4-pass, introducing additional phase advances allows greater improvement in I_{th} than x-y coupling

Questions?

Acknowledgment

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Christopher Mayes
David Sagan

References

Hoffstaetter, G. H. and I. V. Bazarov. *Beam-breakup instability theory for energy recovery linacs*. Physical Review Special Topics - Accelerators and Beams, **7** (2004).

Georg H. Hoffstaetter, I. V. B. and C. Song. *Recirculating beam-breakup thresholds for polarized higher-order modes with optical coupling*. Physical Review Special Topics - Accelerators and Beams, **10** (2007).

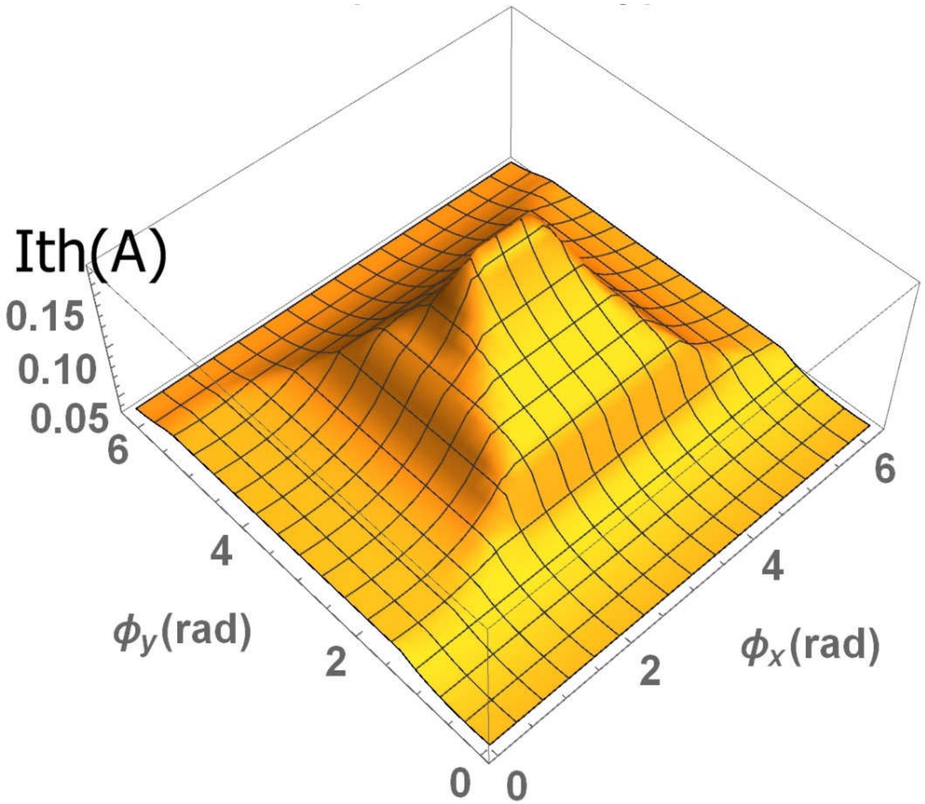
- Bmad manual

THE END

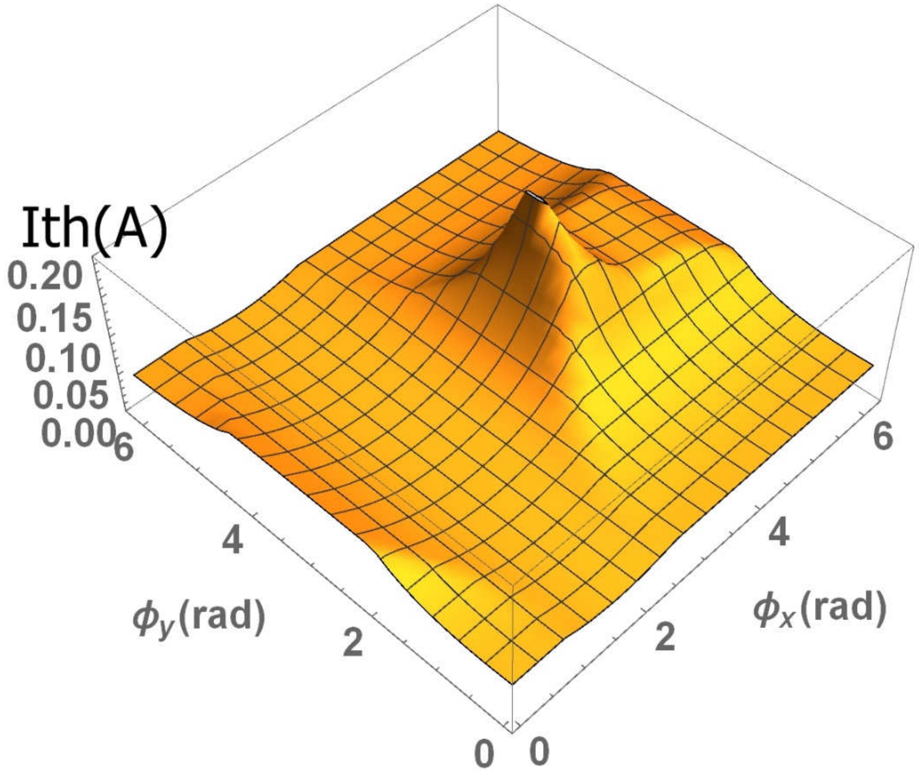
RFC cavity HOMs from Nick Valles

Shape variations in the optimized 7-cell cavity geometry were simulated by adding random errors to each ellipse parameter from a uniform distribution for the error cases of $\pm 1/8$, $\pm 1/4$, $\pm 1/2$ and ± 1 mm. These resulting cavity shapes were tuned cell by cell to 1.3 GHz to ensure field flatness. Subsequently the dipole mode spectrum was calculated up to 10 GHz, using 4 boundary conditions at the at the center plane of the HOM beamline absorbers at the ends of the cavity beamtubes (electric-electric, magnetic-magnetic, electric-magnetic and magnetic-electric) to simulate the superposition of HOMs that are possible for a cavity in a long cavity string.

4-pass decoupled with different HOM assignments…

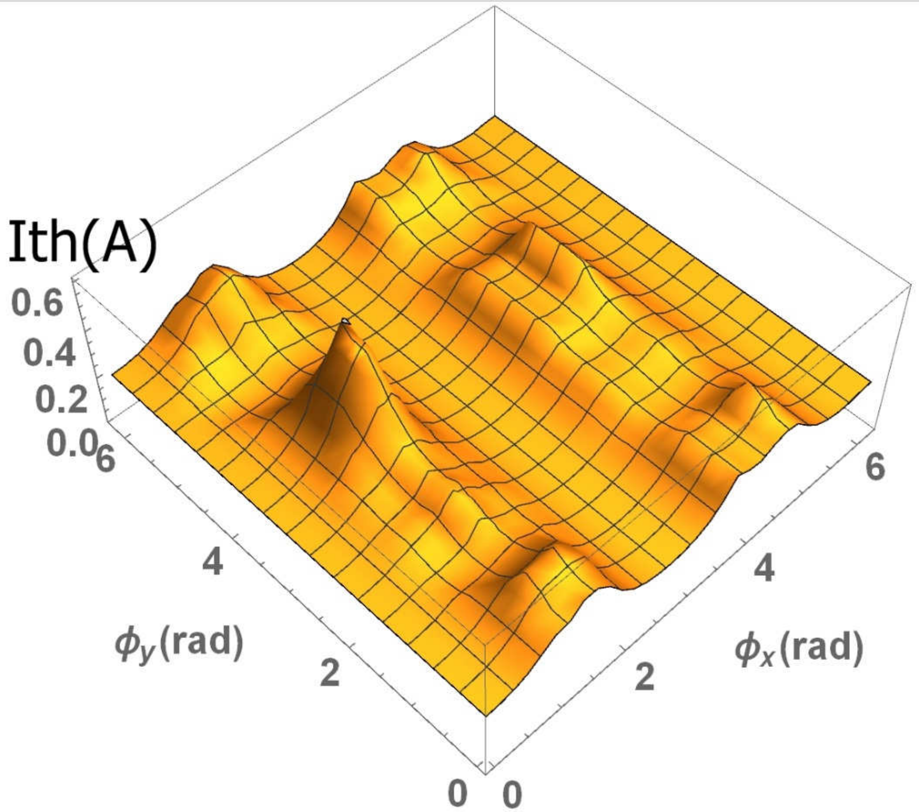


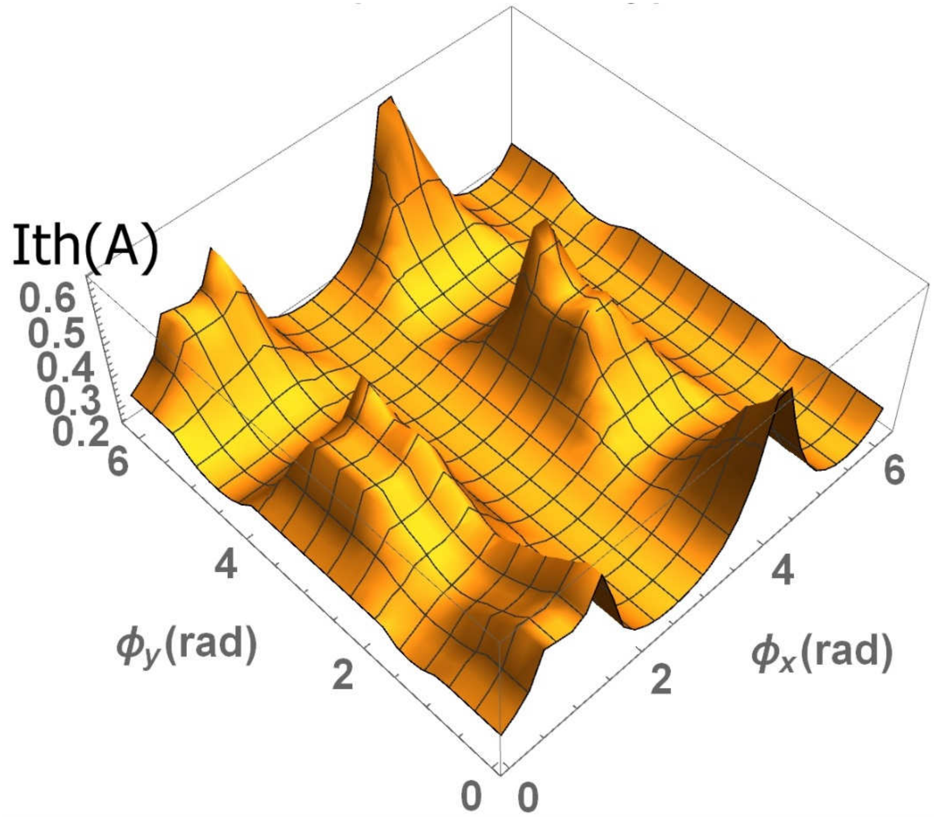
Min = 37 mA
Max = 176 mA
Nominal = 38 mA



Min = 39 mA
Max = 205 mA
Nominal = 49 mA

1-pass decoupled with different HOM assignments…

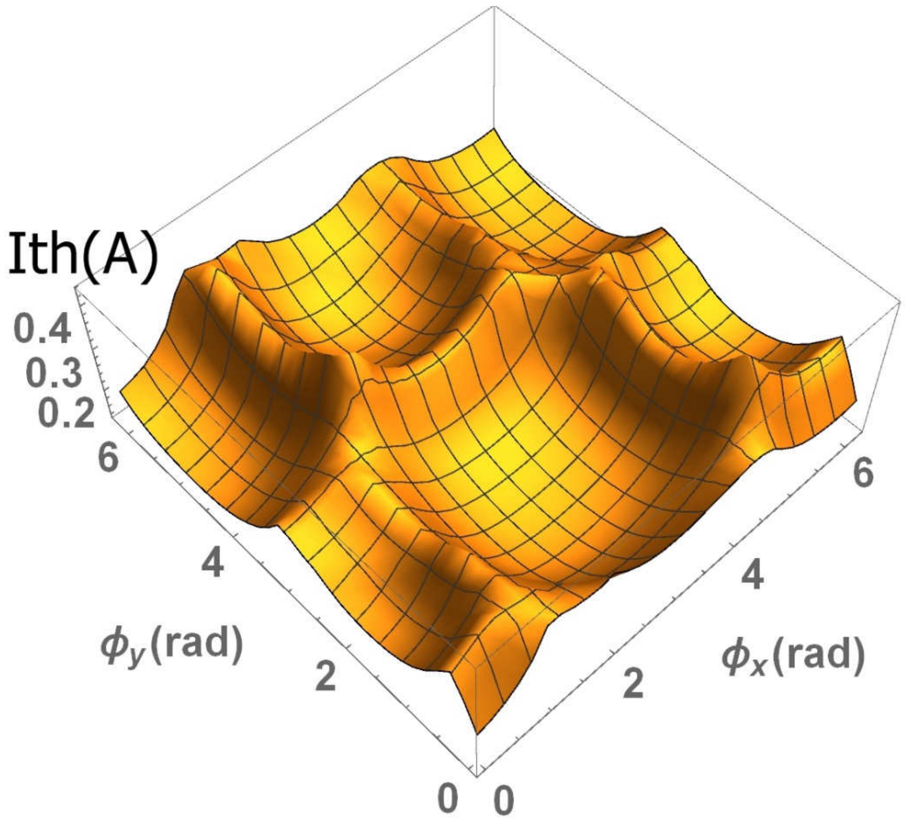




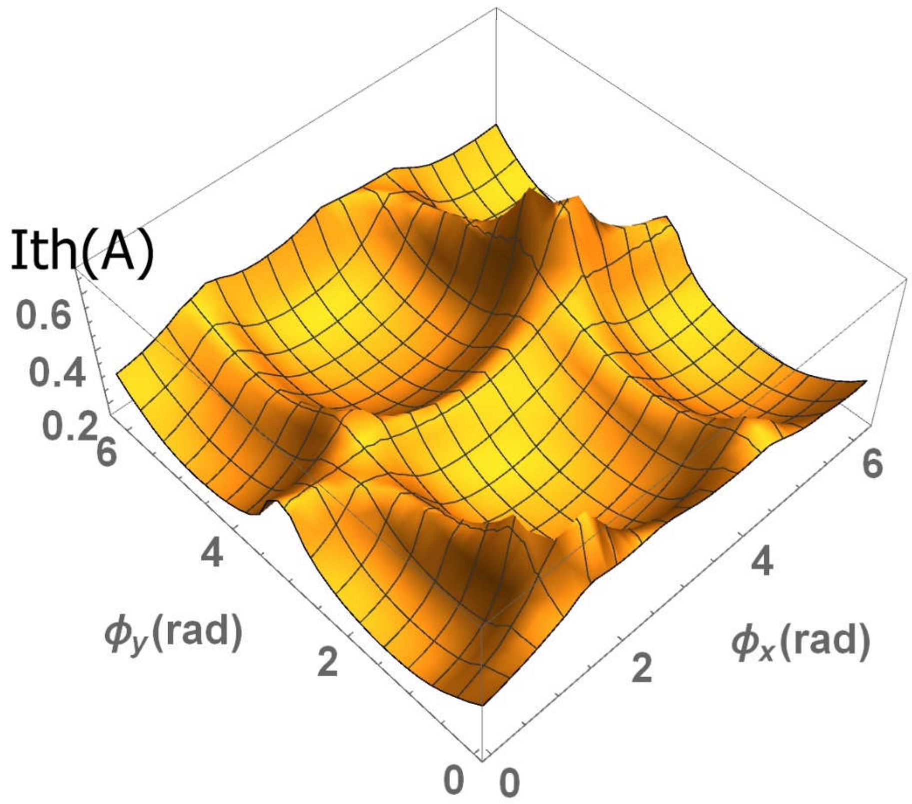
Min = 175 mA
Max = 640 mA
Nominal = 218 mA

Min = 165 mA
Max = 595 mA
Nominal = 248 mA

1-pass coupled with different HOM assignments…

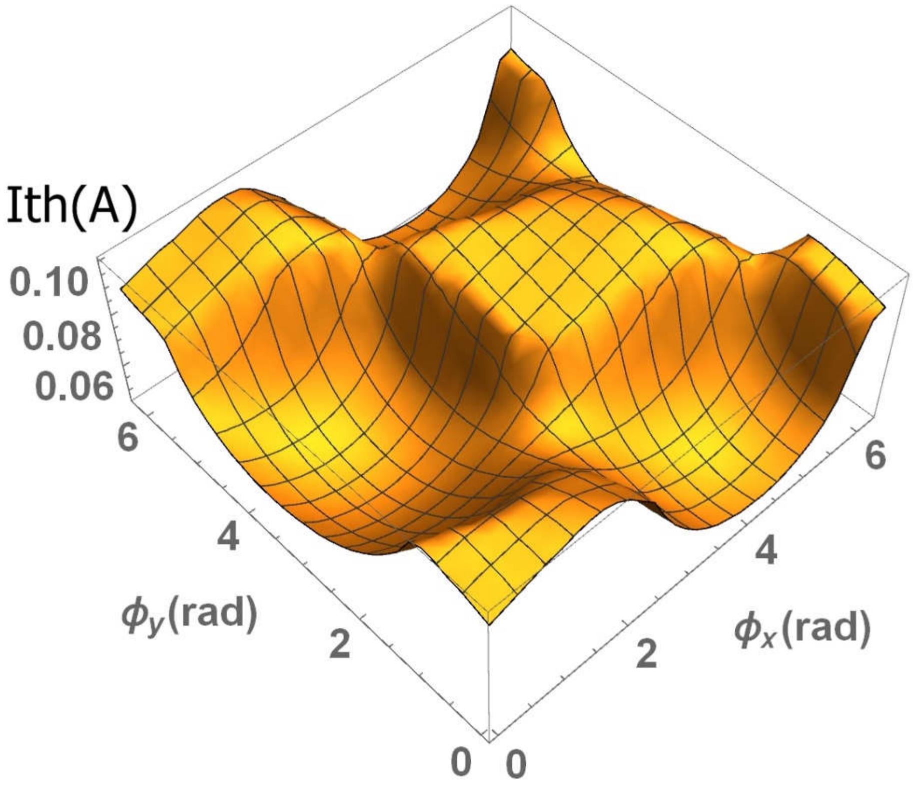


Min = 136 mA
Max = 436 mA
Nominal = 218 mA

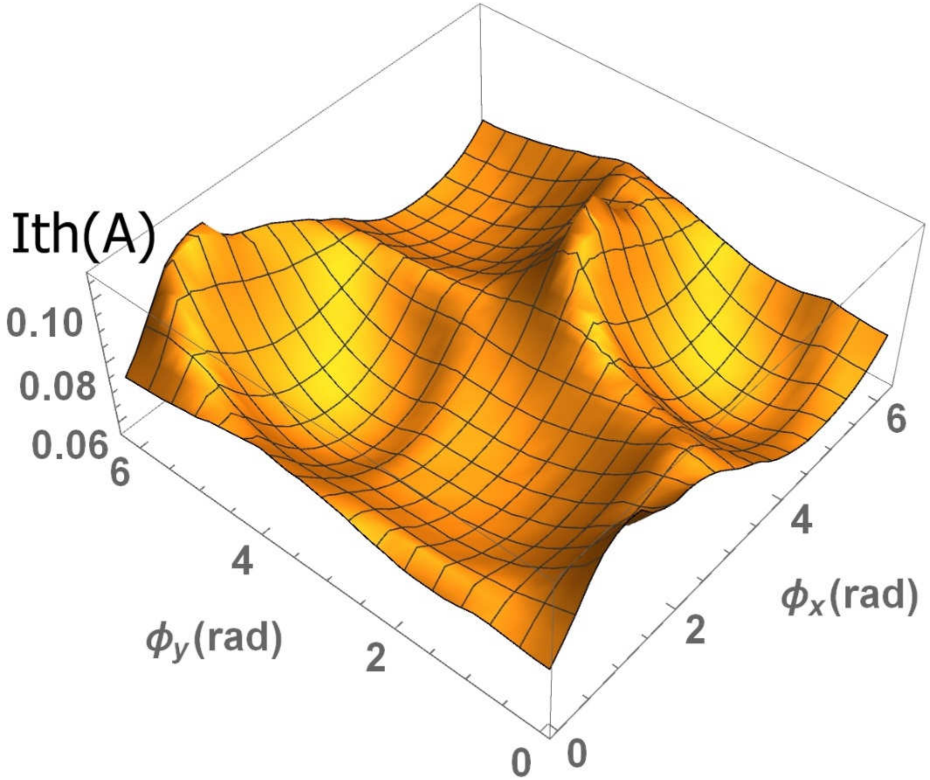


Min = 166 mA
Max = 678 mA
Nominal = 248 mA

4-pass coupled with different HOM assignments…



Min = 47 mA
Max = 100 mA
Nominal = 38 mA



Min = 57 mA
Max = 107 mA
Nominal = 49 mA